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A Turbulent Boundary Layer with Mass Addition, Combustion, and Pressure Gradients

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A subsonic turbulent boundary layer with mass addition and combustion is studied to investigate the effects of combustion on the velocity profiles in constant pressure and accelerating flows. Particular attention is given to determining 1) the extent to which combustion alters the flow and 2) the mechanisms whereby combustion interacts with the flowfield. The experimental results obtained in this study demonstrate that combustion significantly alters the velocity profiles in both constant pressure and accelerating flows. The wall velocity gradients in the combusting flows differ markedly from those of noncombusting flows and show a definite dependence on the pressure gradient. Additionally, the velocity in the flame region of an accelerating flow actually exceeds the freestream value. Analytical results indicate that the experimentally observed changes in the velocity profiles are attributable to the temperature dependence of the local mean density and molecular viscosity. A method of calculating the velocity in a combusting turbulent boundary layer is also presented. Calculated and experimental velocity profiles are compared.

Nomenclature

A = van Driest parameter, $A(\tau_w, \rho_w v_w, dp/dx)$
f' = velocity ratio u/u_e
F = mass injection ratio $\rho_w v_w / \rho v$
p = pressure
u,v = flow velocities
x,y = physical coordinates
δ = boundary-layer thickness
δ_F = flame zone position
ϵ = eddy viscosity
η,ξ = transformed coordinates
θ = momentum thickness
μ = molecular viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress

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Index Category: Boundary Layers and Convective Heat Transfer—Turbulent; Subsonic and Supersonic Airbreathing Propulsion.

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Introduction

COMBUSTING boundary layers are encountered in a number of flow environments of current interest and the need to understand heat, mass, and momentum transfer processes in these environments has stimulated a number of theoretical and experimental studies of this phenomenon. However, review of the literature[1] has indicated that the bulk of these studies have considered laminar boundary layers and only relatively few have investigated turbulent boundary layers.

Additionally, the few investigations which have treated the turbulent cases have limited themselves to constant pressure flows and have not included the effects of pressure gradient interactions in their analysis or experiments.[2,3] Study of turbulent boundary layers, including consideration of the effects of pressure gradients is, however, particularly important as most combusting boundary layers will be turbulent and subject to a nonzero pressure gradient. It is, therefore, the purpose of this paper to study the interaction between the combustion process and the velocity field in both constant pressure and accelerating flows and to investigate the mechanism whereby this interaction takes place. Both experimental and analytical results are presented.

The experimental environment studied was a turbulent boundary layer along porous plates set in the wall of a subsonic wind tunnel. A mixture of nitrogen and hydrogen was injected through the porous plates into the boundary layer. The turbulence in the boundary layer then mixed the nitrogen-hydrogen injectant with air entrained from the freestream. The result of this mixing was a thin combustible zone within the boundary layer which was ignited by external means. Once initiated, the combustion process was self-sustaining.

The analytical portion of this study centered on an investigation of the manner in which combustion interacts with the flowfield. Calculated velocity profiles were obtained through the numerical solution of a transformed differential momentum equation. These velocity profiles were then compared with the experimental velocity profiles. The calculated and experimental velocity profiles agreed in their indication of the effects of combustion on the flowfield. This analysis was carried out on a Univac 1108 computer using a number of standard numerical routines.

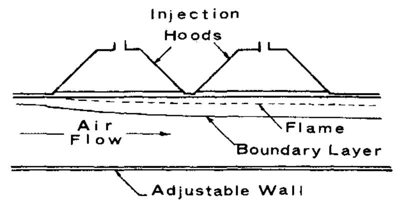
Experimental Study

Experimental Facility and Test Procedures

The wind tunnel used in this study was a low-turbulence, open-loop tunnel similar in design to that of Wooldridge and Muzzy.[2] The tunnel had a controlled flow velocity range of from 5 to 150 fps. The test section was 8 ft in length with a 7- × 7-in. cross section. A schematic drawing of the test section is shown in Fig. 1a. Along a portion of the test section, two porous inconel plates, each 6.5 × 12.5 × 0.5 in. were set in the top wall. The nitrogen-hydrogen mixture was injected through these plates into the boundary layer. The hydrogen and nitrogen flow were metered and controlled separately and by this means, the composition and total flow rate was controlled. The injectant gases were mixed in a plenum chamber prior to entering the injection hoods. The bottom wall of the test section was mounted on a series of screw jacks that were adjusted to establish the desired axial pressure gradient.

Total pressure, static pressure, and temperature variations through the boundary layer were measured at several axial stations along the porous plates. The majority of these data were taken with a multihead probe. This probe consisted of four separated probes mounted so that the sensing elements lay in the same horizontal and vertical plane but were spaced laterally to avoid mutual interference (Fig. 1b). The fourth probe in this configuration was used to draw samples of the combustion mixture for the determination of concentration profiles. However, due to difficulties with the mass spectrographic equipment available to analyze gas samples at the time of these experiments, such data were not obtained. The experimental concentration profiles obtained by Wooldridge and Muzzy[2] and Smith[4] in similar experiments were used to correct the present data for the effects of variable molecular weight. These corrections were small, however, with the maximum velocity correction being only approximately 10%.

Fig. 1a Test section.



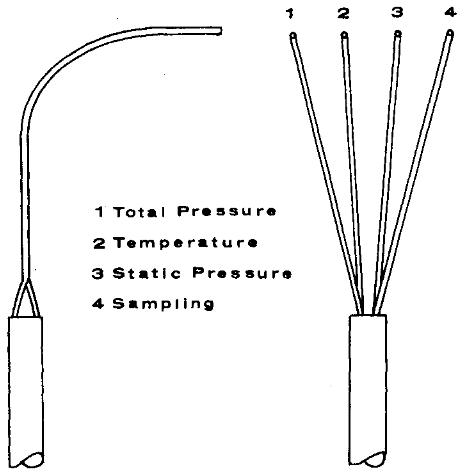


Fig. 1b Multihead probe.

The multihead probe configuration was chosen to minimize the size of the individual probes but yet provide simultaneous measurements. Wooldridge and Muzzy[2] reported successful use of a similar probe design. Measurements with conventional pressure and temperature probes were also made to check the results obtained with the multihead probe.[1]

Measurements of the dynamic pressure and temperature were made at from 30 to 40 points in the boundary layer at each axial station. Similar measurements were made to determine if there was a variation in the static pressure through the boundary layer. Results obtained with a micromanometer indicated only a negligible (± 0.004 in H_2O) variation in the normal static pressure. The axial static pressure gradient was measured using a series of wall static pressure taps and a differential manometer. The freestream dynamic pressure was monitored continously during each test run.

For the experiments being presented here, measurements were made at four positions along the porous plates at a mass injection rate of $F = 3.1 \cdot 10^{-3}$. The injectant was a mixture of nitrogen with 3.8% by mass hydrogen. The freestream velocity of the constant pressure test was 33.6 fps while the velocity in the accelerating flow varied from approximately 32 to 36 fps over the length of the porous plates.

Boundary-Layer Profiles

Profiles of velocity, density and temperature and the boundary-layer dimensions have been determined from the measured temperature and dynamic pressure profiles. These profiles of mean properties, along with the measured axial static pressure gradient, provide an effective means of observing the influence of combustion on a turbulent boundary layer.

Constant pressure flow

The measured velocity profiles at two axial stations in a constant pressure flow are shown in Fig. 2. It is immediately apparent from these curves that the combusting layer velocity profiles differ considerably from those in an isothermal layer. The most striking evidence of this is the inflection in the profiles in the vicinity of the flame zone. It should be noted that this inflection becomes stronger as the burning

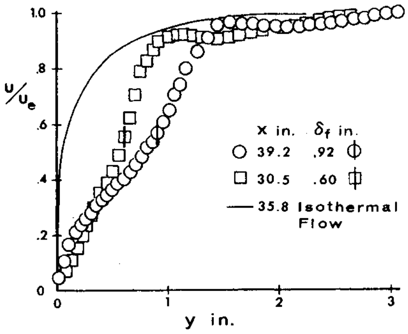


Fig. 2 Constant pressure flow velocity profies.

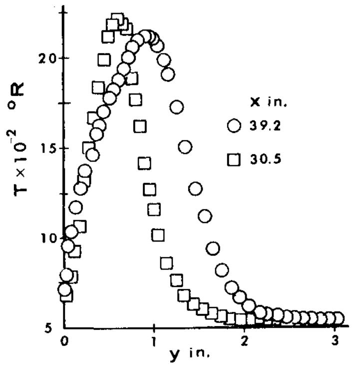


Fig. 4 Constant pressure flow temperature profiles.

boundary layer develops. Examination of the profiles at all four stations[1] indicates that although the boundary layer is perturbed by the initiation of mass addition and combustion the flow appears to approach an "equilibrium" condition at the $x = 39.2$-in. station. Conditions, including wall temperature, were uniform over the second injection plate and the last velocity profile is considered representative of equilibrium flow profiles.

The actual extent of the difference between the velocity profile in a combusting layer and that of an isothermal layer is illustrated by Fig. 3. It may be observed from these data that the two profiles differ markedly in the wall region. This difference in wall velocity gradient indicates a potentially large difference in skin friction between the two flows.

Representative temperature profiles for the constant pressure flow are presented in Fig. 4. These profiles are sharply peaked, indicating a relatively narrow flame zone, with temperatures dropping off rapidly on either side of the maximum. When plotted against a normalized distance y/δ_F the temperature profiles tend to collapse toward a single curve. This trend is illustrated in Fig. 5 where the temperature profiles are plotted in this fashion. The parameter δ_F is the distance from the wall to the maximum temperature point. The position of the flame zone is a function of both the boundary-layer thickness and the fuel species (hydrogen) concentration at the wall. The velocity profiles, normalized in this manner, also tend toward a single curve.[1]

Accelerating flow

When the boundary layer is subjected to a favorable axial pressure gradient, the difference between the combusting and noncombusting velocity profiles is even more pronounced. Figure 6 shows velocity profiles at two stations in a combusting turbulent boundary layer with a small favorable pressure gradient, $dp/dx = -0.16$ psf/ft. Here it is observed that the velocity in the vicinity of the flame zone actually exceeds the freestream value. This velocity overshoot is caused by the interaction of the axial pressure gradient and the density gradient with the velocity field.

The flame zone position δ_F is again an effective normalization parameter in the accelerating flow. This is demonstrated for the temperature data in Fig. 7. The temperature profiles for this case are quite similar to those for the constant pressure flow. The velocity overshoot shown by the data of Fig. 6 indicates a slight increase with increasing axial distance. However, flow analysis indicates that this is to be expected with a constant pressure gradient.

Figure 8 shows a comparison of the velocity profiles in a turbulent boundary layer with mass addition and a small favorable pressure gradient with and without combustion. This figure illustrates the strong influence that the combination of combustion and a pressure gradient has on the velocity profile. It is again noted that velocity gradient at the wall is less in the combusting flow than in the noncombusting flow. It is also of interest to compare the velocity profiles for combusting flow with and without a small favorable pressure gradient. This is done in Fig. 9. These data indicate that for the small pressure gradient of the present experiment the main difference in the profiles is the velocity overshoot which occurs in the accelerating flow. However, for more strongly accelerating flows both the velocity overshoot and the wall velocity gradient will increase. The velocity overshoot phenomenon observed in these experiments has not been previously measured or predicted analytically for a turbulent boundary layer. Chen and Toong,[5] however, did analytically predict that such a velocity overshoot would occur in an accelerating laminar boundary layer with mass addition and combustion.

These experimental data demonstrate that a strong combustion-velocity field interaction exists. The most significant effects of this interaction being its influence on the velocity gradient at the wall and the velocity overshoot which occurs in the flow with a favorable pressure gradient.

Analysis

Several authors[6-8] have pointed out that the development of a general theory for the variable density turbulent boundary layer will be impossible until a great deal more has been learned about the nature of turbulent transport mechanisms. Consequently, most analytical studies of these flows concern themselves with semiempirical parametric correlations or with the development of approximate mathematical models. The mathematical model approach is used in the present study. Consideration of complete solution of the equations of motion for noncombusting flow[8-10] indicate that such solutions for

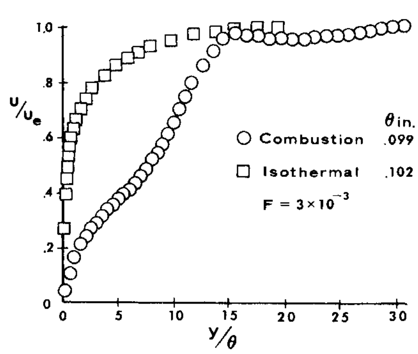


Fig. 3 Comparison of velocity profiles with and without combustion.

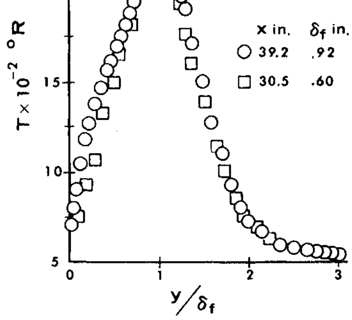


Fig. 5 Normalized temperature profiles.

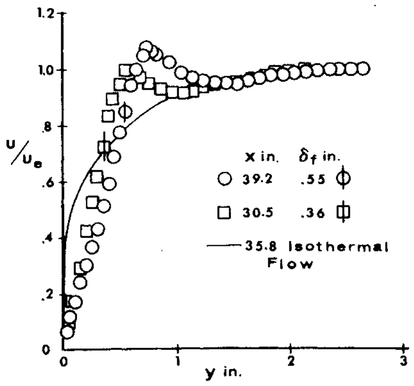


Fig. 6 Accelerating flow velocity profiles.

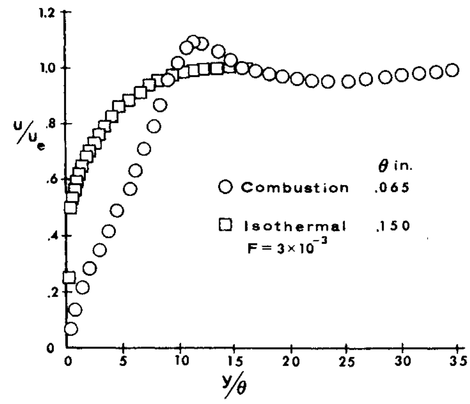


Fig. 8 Comparison of velocity profiles with and without combustion.

the flow under study here would require a very extensive computational effort. However, it is possible to make considerable progress toward understanding the experimentally observed interaction between combustion and the velocity field by means of a more limited analysis.

The following analysis presents an approximate method for numerical solution of the momentum equation through the use of experimental temperature and density profiles and a modified eddy viscosity distribution. The resulting velocity profiles are then compared with the experimentally observed velocity profiles to gain an understanding of the interaction between the combustion-induced density and temperature gradients and the velocity field. Further, these comparisons indicate the extent of combustion-induced changes in the velocity field and suggest the mechanism whereby the interaction occurs.

Governing Equations

The derivation of the equations of motion which describe a compressible turbulent boundary layer may be found in a number of sources.[6,8,10] For a two-dimensional flow, the axial momentum equation may be written

$$\langle\rho\rangle\langle u\rangle\partial\langle u\rangle/\partial x + (\langle\rho\rangle\langle v\rangle + \langle\rho'v'\rangle)\partial\langle u\rangle/\partial y = -d\langle p\rangle dx + (\partial/\partial y)(\langle\tau_{xy}\rangle - \langle\rho\rangle\langle u'v'\rangle) \quad (1)$$

where the angular brackets refer to mean motion and the prime to turbulent fluctuation. As there is no exact theoretical means of describing the turbulent stress term, $\langle u'v'\rangle$, which appears in this equation, an approximate or empirical means of handling it must be found.

Turbulent transport properties

Two alternative approaches have been applied to obtaining turbulent transport properties in boundary layer flows. One method, developed by Bradshaw and Ferris,[11] transforms the differential equation describing turbulent kinetic energy into an equation for turbulent shear stress. This equation then provides the additional input necessary to solve the conservation equations. The second approach used by Cebeci and Smith[9] and Mellor and Herring,[10] and which will be followed here, describes the turbulent transport properties in terms of mean flow quantities through a turbulent eddy viscosity defined by the Boussinesq equation

$$\langle\rho\rangle\epsilon\partial\langle u\rangle/\partial y = -\langle\rho\rangle\langle u'v'\rangle \quad (2)$$

It is assumed that the eddy viscosity follows the relationship

$$\epsilon_i = l^2\partial u/\partial y \quad (3)$$

in the inner region of the boundary layer and

$$\epsilon_0 = k_2 u_e \delta_k^* \gamma \quad (4)$$

in the outer region. In these expressions l is the mixing length and γ is Klebanoff's[13] intermittency factor.

The constants k_1 and k_2 are taken to be 0.40 and 0.0168, respectively, and l, δ_k^* and γ are defined as

$$l = k_1 y[1 - \exp(-y/A)], \quad \delta_k^* = \int_0^\infty (1 - \langle u\rangle/u_e)dy$$

$$\gamma = [1 + 5.5(y/\delta)^6]^{-1}$$

The inner and outer regions of the boundary layer are delineated by the point at which $\epsilon_i = \epsilon_0$.

The axial momentum equation expressed in terms of this eddy viscosity is

$$\langle\rho\rangle\langle u\rangle\partial\langle u\rangle/\partial x + (\langle\rho\rangle\langle v\rangle + \langle\rho'v'\rangle)\partial\langle u\rangle/\partial y = -\partial\langle p\rangle/\partial x + (\partial/\partial y)[\langle u\rangle + \langle\rho\rangle\epsilon]\partial\langle u\rangle/\partial y \quad (5)$$

The mass, energy, and species equations may be handled in a similar manner. The solution of these equations is facilitated by re-expressing them in a nondimensional form.

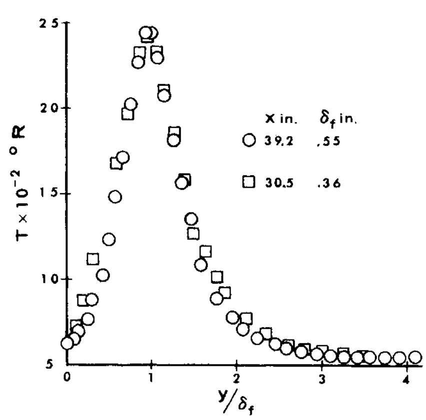


Fig. 7 Accelerating flow temperature profiles.

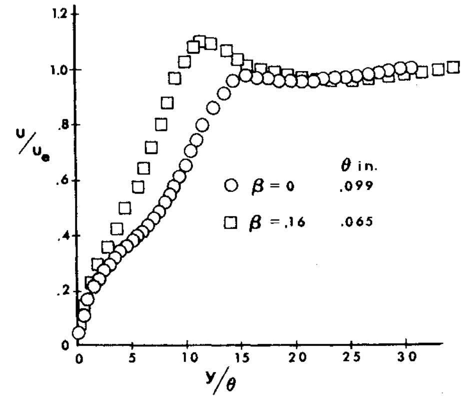


Fig. 9 Comparison of combusting flow velocity profiles with and without a pressure gradient.

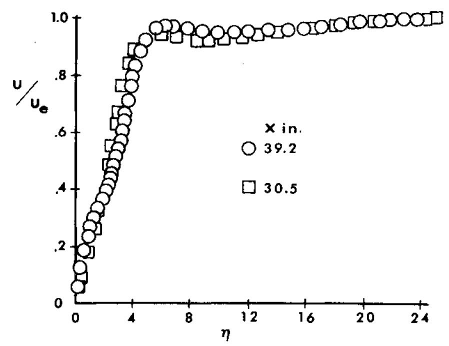


Fig. 10 Transformed velocity profile—constant pressure flow.

Transform Equations

Following the work of Cebeci et al.[9,14] transformations of the Doronitsyn type are utilized. The transformed coordinates are

$$\xi = \int_0^x \rho_e u_e \mu_e dx, \quad \eta = \frac{\rho_e u_e}{(2\xi)^{1/2}} \int_0^y \left(\frac{\rho}{\rho_e}\right) dy \tag{6}$$

Employing these transformations and defining a nondimensional stream function $f(\xi,\eta)$ such that

$$f'(\xi,\eta) = u/u_e$$

the conservation equations become§

Momentum

$$\{(\rho\mu/\rho_e\mu_e)[1 + \epsilon/\nu]f''\}' + ff'' + \beta(\rho_e/\rho - [f']^2) = 2\xi(f'\partial f/\partial\xi - f''\partial f/\partial\xi) \tag{7}$$

where the primes now indicate differentiation with respect to η and β is defined as

$$\beta = (2\xi/u_e)du_e/d\xi$$

Numerical Analysis

Laufer[8] has observed from the experimental work of Kistler[12] that for flows with an edge Mach number less than 5, the momentum equation is coupled to the energy equation only through the spatial variation of the mean density and viscosity. He further states that this "implies that the density variation has a kinematic or volumetric rather than dynamic effect on the velocity field in a compressible turbulent medium." The magnitude of density and temperature variations observed in the present flow are comparable to those of the experimental work[12] cited by Laufer.[8] Therefore, it will be assumed that in the present case, the momentum equation is coupled to the energy and species equations only through the mean density and viscosity. Mean density and viscosity are in turn primarily dependent upon the local temperature. The similarity of the experimental temperature profiles from one axial position to another, as demonstrated by Figs. 5 and 7, and the consistency of the present temperature data with that of a previous investigation[2] suggests that it would be reasonable to uncouple the momentum equation from the energy equation by determining the density and viscosity from normalized experimental temperature profies. In this way the momentum equation may be solved for the velocity profiles without requiring the simultaneous solution of the energy and species equations. This simplifies numerical procedures and significantly reduces computation time.

A second simplification which will be made is associated with transformations carried out in the previous section. The essential purpose of the transformation process is to minimize the axial variation of the transformed boundary-layer thickness and mean property profiles. In the limit these transformations lead to "similar" profiles with no axial variation and the transformed equations reduce to ordinary rather than partial differential equations. Although the transformations utilized here do not lead to similar profiles in turbulent flow, they do significantly reduce the axial variations in the velocity profiles. This is demonstrated by the data of Fig. 10 where the velocity ratio is plotted against the transformed normal coordinate. On the basis of this observation and the approximate nature of the present analysis, the nonsimilar terms in Eq. (7) were neglected. The momentum equation then becomes a total differential equation of the form

$$[\phi f'']' + ff'' + \beta[\rho_e/\rho - (f')^2] = 0 \tag{8}$$

where

$$\phi = (\rho u/\rho_e u_e)(1 + \epsilon/\nu)$$

The parameter β is a constant for a given axial station while ϕ is a function of the normal coordinate $\phi(\eta)$. Equation (8) may be conveniently solved by numerical means once the variation of ϕ with η has been specified.

Results

The momentum equation, in the form of Eq. (8), has been solved numerically on a Univac 1108 digital computer using a boundary value program developed by Vreeke.[15] Prior to using this program to calculate the velocity profiles in combusting flow, its operation was checked by calculating velocity profiles for a number of flows for which other analytical or experimental results were available. The laminar flow Falkner-Skan equation was solved and results compared with those of Hartree.[16] Agreement was excellent. Calculations were also made for the laminar flow variable density case [Eq. (8) with $\phi = 1$ and $\rho = \rho(\eta)$] and results compared with those of Chen and Toong,[15] again with good agreement. As a final check, Eq. (8) was used to calculate the velocity profile in an isothermal turbulent boundary layer along a flat plate. For this calculation an experimental eddy viscosity distribution derived from the experimental results of Klebanoff[13] and Schubauer[17] was used to define ϕ. The resulting velocity profile is compared with Klebanoff's experimental results and with the data of AlSaji[18] in Fig. 11. This comparison indicates the extent of validity of the calculation procedures and similarity assumption. The experimental results of AlSaji were obtained in the same test facility being used in the present experiment. It should be noted that this method of determining velocity profiles is qualitative and although it yields normalized velocity profiles that agree well with normalized experimental profiles, it is not structured to predict quantitative values such as boundary-layer thickness. The comparison of normalized profiles does, however, provide an appreciable amount of insight into the combustion-velocity field interaction under study.

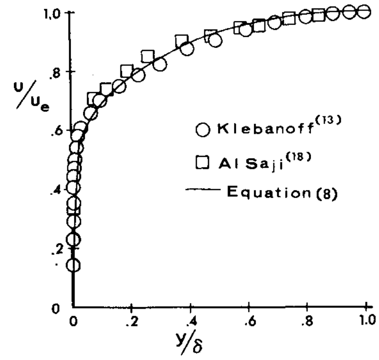


Fig. 11 Velocity profile in an incompressible turbulent boundary layer.

§ The angular brackets and prime notation will be dropped at this point as the equations are now expressed in terms of mean quantities only.

Constant pressure flow with combustion

In calculating velocity profiles for the combusting turbulent flow cases values of ρ and μ were determined from experimental data so that the momentum equation might be solved independently. This was accomplished in a straightforward manner and the resulting density and viscosity profiles put into the computer program in tabular form. However, difficulty was encountered in determining values for the eddy viscosity. Recalling the results presented in the previous section which indicated that combustion should affect the eddy viscosity in a "kinematic" sense, it was concluded that experimental eddy viscosity values applicable to isothermal flow might be adjusted to account for the effects of mass injection and combustion and used in the present calculations. For the outer region the eddy viscosity was tabulated as $\epsilon = \epsilon(\eta/\eta_e)$ and then it was assumed that

$$[\epsilon/u_e\delta_k{}^*]_{\text{comb}} = [\epsilon/u_e\delta_k{}^*]_{\text{exp}} \tag{9}$$

or

$$\epsilon_{\text{comb}} = [u_e\delta_k{}^*]_{\text{comb}} \cdot [\epsilon/u_e\delta_k{}^*]_{\text{exp}}$$

at corresponding values of η/η_e. In the inner region eddy viscosity expression, Eq. (3), the parameter A, and thus the mixing length l, is very difficult to define for the present environment. Because of this difficulty another means of defining the eddy viscosity in this region was sought. Smith,[4] in his study of mass injection in turbulent boundary layers, developed a numerical method for determining the eddy visvosity in the inner region from experimental velocity, temperature, and concentration measurements. This method was applied to the experimental data of Figs. 2 and 4 to obtain values of ϵ for the inner region. Smith's method is only applicable in the inner region as it depends on the numerical differentiation of experimental data and looses accuracy when successive differences become small. Additionally it was noted that the application of the scaling factor indicated by Eq. (9) to the experimental values from isothermal flow in the inner region resulted in values for ϵ very nearly equal those obtained from Smith's method. With these values of ϵ and the experimentally determined values of ρ and μ the viscosity parameter ϕ was calculated. The distribution of these values of ϕ is compared with the isothermal ϕ distribution derived from the data of Klebanoff and Schubauer in Fig. 12. The values ϕ obtained by this method agree with those from Eq. (4) throughout the outer region.

The ϕ values shown in Fig. 12 have been used in conjunction with Eq. (8) to predict the velocity profile in a constant pressure combusting turbulent boundary layer. The results of this calculation are compared with experimental data in Fig. 13. The agreement between calculated and experimental values is good with the predicted velocity profile showing

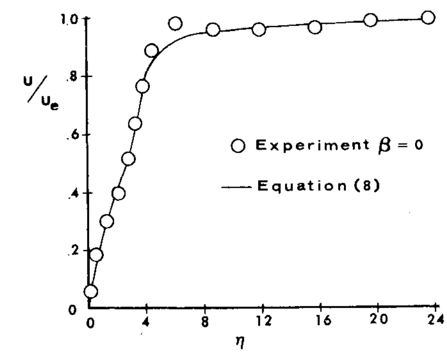


Fig. 13 Velocity profile in a constant pressure flow with combustion.

both the lower velocity gradient at the wall and the inflection point in the flame region. Based on the assumptions made in these calculations, the results clearly indicate that the predominant factor influencing the combusting flow velocity profile is the temperature dependence of ϕ.

Accelerating flow

The same calculation procedure has been applied in an attempt to obtain an understanding of the general characteristics of an accelerating flow with combustion. In such flows the momentum equation is coupled to the temperature dependent density gradient through the term $[\rho_e/\rho + (f')^2]$ as well as through ϕ. In calculating velocity profiles from Eq. (8) for accelerating flows the ϕ distribution of Fig. 12 was again used. As it is known that the inner region eddy viscosity is influenced somewhat by the presence of a pressure gradient, this choice is not entirely satisfactory. However, as the temperature dependence of ϕ is the predominant factor, the calculated velocity profiles should be representative.

Velocity profiles for $\beta = 0.15$, 0.30, 0.60, and 1.0 have been calculated from Eq. (8) and are shown in Fig. 14. The experimental data for $\beta = 0.16$ is also shown in this figure. Although these experimental data and the calculated profile do not agree well in a quantitative sense there is definite qualitative agreement with both experimental and calculated values showing the velocity overshoot.

Assuming that the calculated velocity profiles of Fig. 14 are representative of those in a combusting turbulent boundary layer subjected to a favorable pressure gradient, a number of interesting observations may be made. First, the extent of the velocity overshoot is a function of the strength of the pressure gradient. Secondly, as a consequence of the velocity overshoot, the velocity gradient in the wall region varies

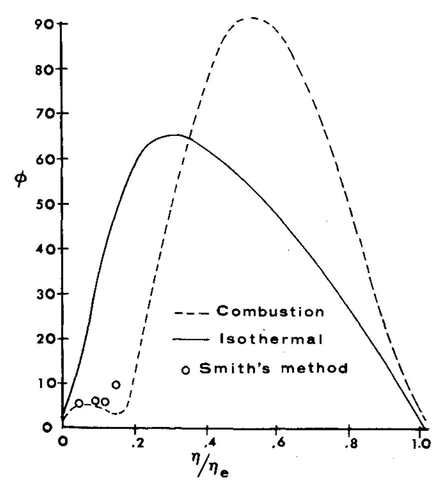


Fig. 12 Viscosity parameter.

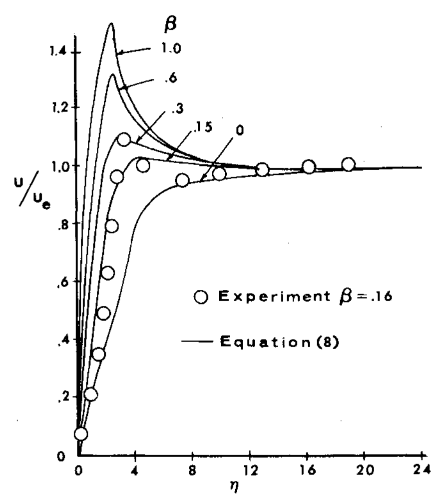


Fig. 14 Velocity profiles in an accelerating flow with combustion.

markedly. For small values of β the wall velocity gradient is less than that for the corresponding isothermal flow while at larger β the combustion flow wall gradient is greater than that in noncombusting flow. These results indicate that the pressure gradient has a significant influence on skin friction.

Conclusions

This investigation was undertaken to study the nature and extent of the interaction between combustion-induced temperature and density gradients and the velocity field in constant pressure and accelerating boundary-layer flows. The results of both experimental and analytical studies have indicated the following.

1) The temperature and density gradients interact with the velocity field in such a way as to significantly alter the velocity profiles from those observed in isothermal flow. In constant pressure flow it has been determined that the velocity gradient at the wall is markedly reduced by combustion. In accelerating flow it has been determined that a velocity overshoots occurs within the boundary layer and that as a consequence of this overshoot the wall velocity gradient is strongly dependent on the pressure gradient.

2) The observed combustion-induced changes in the velocity field are attributable, in the main, to the temperature dependence of mean properties. Additionally, the agreement between the analytical and experimental results validates the assumption that the eddy viscosity is kinematic in nature.

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